

**MATH 464 (THEORY OF PROBABILITY)
HOMEWORK 5**

SPRING 2019
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- (1) Given a random variable X with probability mass function

$$f_X(k) = \frac{c}{k(k+1)}, \text{ for } k = 1, 2, \dots$$

- (a) Find the value of c .
(b) Find $\mathbb{E}(X)$.
- (2) Find the mean of $X \sim \text{Geometric}(p)$.
- (3) Let X be a discrete random variable with a geometric distribution whose mean is 2.
(a) Find $\mathbb{P}(X \geq 6 | X > 4)$, your answer should not have a summation.
(b) Let $Z = X + 3$. Find the mean and variance of Z .
- (4) Can we find a random variable X with $\mathbb{E}(X) = 4$ and $\mathbb{E}(X^2) = 15$? Explain!
- (5) For any random variable X , $M_X(t) := \mathbb{E}(e^{tX})$ is the *Moment generating function* (to be discussed in Chapter 4). Find $M_X(t)$ if X is a discrete random variable from
(a) Bernoulli(p) (b) $b(n, p)$ (c) Geometric(p) (d) Poisson(p).
- (6) Give an example of a discrete random variable X such that $\mathbb{E}(X^n) = 3^{n-1}$ for $n = 1, 2, \dots$. Then find $\text{Var}(X)$, and $\text{Var}(X^2)$.
- (7) The joint probability mass function of the random variables X and Y is given as $f_{X,Y}(x, y) = c(x + y)$ for $x, y = 0, 1, 2, 3$.
(a) Find the value of c .
(b) Find $\mathbb{P}(X > Y)$.
(c) Find $\mathbb{P}(X + Y = 3)$.